RA 08

4.3 Properties of functions

$f: X \to Y$

- A function f is <u>one to one</u> or injective if $x_1 \neq x_2$ implies that $f(x_1) \neq f(x_2)$.
  - that is, f maps different elements in X to different elements in Y.
- A function $f: X \to Y$ is <u>onto</u> or <u>surjective</u> if the range of f is equal to the target Y.
  - that is, for every $y \in Y$, there is an $x \in X$ such that $f(x) = y$.

- A function is <u>bijective</u> if it is both one-to-one and onto.
  - ↳ Also called a <u>bijection</u>
  - ↳ Also called a <u>one-to-one correspondance</u>

- If $f: D \to T$ is <u>onto</u> then $|D| \geq |T|$
- If $f: D \to T$ is <u>one-to-one</u> then $|D| \leq |T|$
- If $f: D \to T$ is a bijection, then $|D| = |T|$

4.4 The Inverse of a Function

- If a function $f: X \to Y$ is a bijection, then the <u>inverse</u> of f is obtained by exchanging the first and second entries in each pair f.
  - $f^{-1} = \{(y, x) : (x, y) \in f\}$
- f^{-1} is a well defined function if every element in Y is mapped to exactly one element in X.

★ • A function f has an inverse if and only if f is a bijection.

4.5 The Composition of Fractions

- f and g are two functions, where $f: X \to Y$ and $g: Y \to Z$. The composition of g with f, denoted $g \circ f$, is the function $(g \circ f): X \to Z$

- Composition is Associative

- The identity function always maps a set onto itself.
  - The identity function of A, denoted $I_A : A \to A$, is defined as $I_A(a) = a$, for all $a \in A$.

- If $f: A \to B$ has an inverse, then f composed with f^{-1} is the identity function.
  - Let $f: A \to B$ be a bijection. Then $f^{-1} \circ f = I_A$ and $f \circ f^{-1} = I_B$

5.1 An Introduction to Boolean Algebra

- 1 corresponds to T
- 0 corresponds to F

- Boolean Multiplication is denoted by "$\cdot$"
  - Think of replacing "$\cdot$" with "$\wedge$"

Boolean $\cdot$
$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$1 \cdot 0 = 0$
$1 \cdot 1 = 1$

- Boolean Addition is denoted by "+"
  - Think of replacing "+" with "$\vee$"

Boolean +
$0 + 0 = 0$
$0 + 1 = 1$
$1 + 0 = 1$
$1 + 1 = 1$

- The complement of an element is denoted with a bar symbol.
  - Think of replacing the bar with "$\neg$"

Boolean Complement
$\overline{0} = 1$
$\overline{1} = 0$

- Boolean Variable : Stores either 0 or 1

- Boolean Expression : can be built up by applying boolean operations to Boolean variables

- Order of Operations for Boolean operations:
  - Multiplication before addition.
  - Complement is applied as soon as the entire expression under the bar is evaluated.
  - Use parentheses to override

- In boolean algebra "=" denotes logical equivalence.

Idempotent Laws	$x+x=x$	$x \cdot x = x$
Associative Laws	$(x+y)+z = z+(y+x)$	$(xy)z = x(yz)$
Commutative Laws	$x+y = y+x$	$xy = yx$
Distributive Laws	$x+yz = (x+y)(x+z)$	$x(y+z) = xy + xz$
Identity Laws	$x+0 = x$	$x \cdot 1 = x$
Domination Laws	$x+1=1$	$x \cdot 0 = 0$
Double Complement Laws	$\bar{\bar{x}} = x$	$\bar{\bar{x}} = x$
Complement Laws	$x+\bar{x}=1$ $\bar{0}=1$	$x\bar{x}=0$ $\bar{1}=0$
DeMorgans Laws	$\overline{x+y} = \bar{x}\bar{y}$	$\bar{x}\,\bar{y} = \bar{x}+\bar{y}$
Absorption Laws	$x+(xy)=x$	$x(x+y)=x$

S.3 Disjunctive and Conjuctive normal form

- A boolean expression that is the sum of products of literals is in disjunctive normal form (DNF).

  - Example: $\bar{x}y\bar{z} + xy + w + y\bar{z}w$
    - ↳ complement only applied to a single variable
    - ↳ No addition within term

- Every Boolean expresson can be represented in DNF.

- A boolean expression that is a product of sums of literals is in <u>conjunctive normal form.</u> (CNF)

  - Each term in the product that is the sum of literals is called a <u>clause</u>.

- Example: $(\bar{x} + y + \bar{z})(x + \bar{y})(w)(y + \bar{z} + w)$

  ↳ complement only applied to single variables

  ↳ No multiplication within a clause.

  ↳ each item within parantheses is called a clause.